In the name of GOD.

Sharif University of Technology

Stochastic Processes

Fall 2025

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Quiz 6 (15 minutes)	Estimation Theory 1	Date: Dec. 9th

1. (60 Points) Let $(X_1, \dots, X_n)$ be i.i.d. with

$$P(X_i = 1) = \theta, \qquad P(X_i = 0) = 1 - \theta, \qquad 0 < \theta < 1.$$

A student proposes the following three claims about *sufficient statistics* for θ:

1. **Claim A:** $T_1 = \sum_{i=1}^n X_i$ is sufficient for θ.

2. **Claim B:** $T_2 = \sum_{i=1}^n (2X_i - 1)$ is *not* sufficient for θ, because it can take negative values and "does not look like a count".

3. **Claim C:** $T_3 = \mathbf{1}\{\text{at least one } X_i = 1\}$ (i.e., 1 if there is at least one success, 0 otherwise) is sufficient for θ, because it still depends on θ.

For each claim (A), (B), and (C):

- State whether it is **correct** or **wrong**, and
- Give a short explanation in words (no detailed calculations needed).

Solotion:

1. **Correct:** The joint pmf depends on the sample only through the total number of 1's, so $\sum X_i$ is sufficient.

2. **Wrong:** T_2 is a one–to–one transform of $\sum X_i$, so it is also sufficient.

3. **Wrong:** Knowing only whether there is at least one success does not capture how many successes occurred, so it is not sufficient.

2. (40 Points) Let $(X_1, \dots, X_n)$ be i.i.d. random variables with pdf

$$f(x \mid \theta) = \begin{cases} \dfrac{1}{1-\theta}, & \theta < x < 1, \\ 0, & \text{otherwise,} \end{cases} \qquad 0 < \theta < 1.$$

Find a sufficient statistic for θ.

Solution:

Joint pdf:

$$f(\mathbf{x} \mid \theta) = \prod_{i=1}^n \frac{1}{1-\theta} \mathbf{1}_{(\theta,1)}(x_i) = (1-\theta)^{-n} \prod_{i=1}^n \mathbf{1}_{(\theta,1)}(x_i).$$

The product of indicators is

$$\prod_{i=1}^{n} \mathbf{1}_{(\theta,1)}(x_i) = \mathbf{1}_{\{\theta<\min_i x_i<1\}} = \mathbf{1}_{\{\theta<\min_i x_i\}}.$$

So

$$f(\mathbf{x} \mid \theta) = (1-\theta)^{-n}\, \mathbf{1}_{\{\theta<\min_i x_i\}}.$$

Factorization:

$$h(\mathbf{x}) = 1, \qquad g\big(T(\mathbf{x}),\theta\big) = (1-\theta)^{-n}\, \mathbf{1}_{\{\theta<T(\mathbf{x})\}},$$

with

$$T(\mathbf{X}) = \min_{1\le i\le n} X_i.$$

By the factorization theorem,

$$\boxed{T(\mathbf{X}) = \min_{1\le i\le n} X_i \text{ is sufficient for } \theta.}$$