In the name of GOD.

Sharif University of Technology

Stochastic Processes

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Quiz 4 (20 minutes) | Poisson Processes + Point Processes | Date: Aban 27th

Problem 1

(a) **True.** A Poisson process with rate $\lambda = 4$ events per hour has expected number of events

$$\lambda t = 4 \times 0.25 = 1$$

in the interval of length 0.25 hours. Therefore the number of events in $(0, 0.25]$ is Poisson(1).

(b) **True.** Over the interval $[0, 0.5]$, the mean number of events is

$$\lambda t = 4 \times 0.5 = 2.$$

If $N \sim$ Poisson(2), then

$$\mathbb{P}(N \geq 2) = 1 - \mathbb{P}(N = 0) - \mathbb{P}(N = 1) = 1 - e^{-2} - 2e^{-2}.$$

Problem 2

(a) Let W be the number of women and M the number of men entering during the hour. We have

$$W \sim \text{Poisson}(2), \qquad M \sim \text{Poisson}(3),$$

independently. Given the total count $Z = W + M = 3$, the conditional distribution is:

$$W \mid (Z = 3) \sim \text{Bin}\left(3,\ p = \frac{\lambda_{\text{women}}}{\lambda_{\text{men}} + \lambda_{\text{women}}}\right) = \text{Bin}(3, 0.4).$$

Thus

$$\mathbb{P}(W = 1 \mid Z = 3) = \binom{3}{1}(0.4)(0.6)^2.$$

Compute:

$$0.6^2 = 0.36, \qquad 0.4 \times 0.36 = 0.144, \qquad 3 \times 0.144 = 0.432.$$

$$\boxed{\mathbb{P}(\text{exactly one woman} \mid Z = 3) = 0.432.}$$

(b) The total arrival process (men + women) is Poisson with rate

$$\lambda_{\text{total}} = 3 + 2 = 5.$$

Thus the number of people entering in the next hour is

$$N \sim \text{Poisson}(5).$$

The probability of exactly 3 arrivals is:

$$\mathbb{P}(N = 3) = e^{-5} \frac{5^3}{3!}.$$