In the name of GOD.

Stochastic Process

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Hamid R. Rabiee

Homework 2	point process, poisson process, power spectrum	Deadline : 1404/08/25

1. Find the PSD for $X(t)$ if:

$$R(\tau) = \begin{cases} 1 - |\tau|, & |\tau| \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

2. Let $X(t)$ be a white Gaussian noise with $S_X(f) = \frac{N_0}{2}$. Assume that $X(t)$ is input to an LTI system with

$$h(t) = e^{-t}u(t).$$

   Let $Y(t)$ be the output.

   (a) Find $S_Y(f)$.

   (b) Find $R_Y(\tau)$.

   (c) Find $E[Y(t)^2]$.

3. Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ and X_1 be its first arrival time. Show that given $N(t) = 1$, then X_1 is uniformly distributed in $(0, t]$. That is show that

$$P(X_1 \leq x | N(t) = 1) = \frac{x}{t}, \quad \text{for } 0 \leq x \leq t.$$

4. Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ. Find its covariance function

$$C_N(t_1, t_2) = \text{Cov}\big(N(t_1), N(t_2)\big), \quad \text{for } t_1, t_2 \in [0, \infty)$$

5. Arrivals of customers into a store follow a **Poisson process** with rate $\lambda = 20$ arrivals per hour. Suppose that the probability that a customer buys something is $p = 0.30$.

   (a) Find the expected number of sales made during an eight-hour business day.

   (b) Find the probability that 10 or more sales are made in one hour.

   (c) ind the expected time of the first sale of the day. If the store opens at 8 a.m.

6. Ali finds coins on his way to work at a Poisson rate of 0.5 coins/minute. The denominations are randomly distributed:

   - 60% of the coins are worth 1 each
   - 20% of the coins are worth 5 each

- 20% of the coins are worth 10 each.

(a) Calculate the probability that in the first ten minutes of his walk he finds at least 2 coins worth
10 each, and in the first twenty minutes finds at least 3 coins worth 10 each.

(b) Calculate the variance of the value of the coins Ali finds during his one-hour walk to work.

7. Two independent Poisson processes have respective rates 1 and 2.Two players start with fortunes a and b. The game evolves as follows:

   - Each time an event occurs in the first Poisson process, player 2 pays one unit to player 1.
   - Each time an event occurs in the second Poisson process, player 1 pays one unit to player 2.

   The game ends when one player's fortune reaches zero; that player loses. Find the probability that player 1 wins.