In the name of GOD.

Stochastic Process

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Quiz 8 (25) | Hypothesis Testing | Release : Azar. 20th

1. Choose the correct option (no need/marks for explanation). (20 points)

   - A (T-test/Z-test) is preferred for small sample sizes and unknown variance.
     Solution: T-test.
     When conducting a Z-test to evaluate a company's claim about the average daily product count, it is assumed that the sample mean follows a normal distribution, regardless of the underlying distribution of the data. (True/False)
     Solution: True.
   - A Z-test may only reject a claim, but it cannot prove the claim to be true. (True/False)
     Solution: True.
   - In a Z-test, rejecting the Alternative Hypothesis (H1) means the Null Hypothesis (H0) is correct. (True/False)
     Solution: False.

2. Define the p-value in the context of hypothesis testing. (10 points)

   Solution: The p-value in hypothesis testing represents the probability of obtaining a test statistic as extreme as or more extreme than the one observed, assuming that the null hypothesis (H0) is true.

3. A high-tech factory manufactures machines that generate products following a Poisson distribution with rates $\lambda = \{25, 36, 50\}$. Your company purchases one of these machines, and your colleague hypothesizes that it is a machine with a generation rate of $\lambda = 36$. You want to test this hypothesis using a Z-test with a significance level of $\alpha = 0.05$. After tracking the number of products generated over 100 days, you observe a total of 3500 products. Conduct the test and report the results. (80 points)

   Solution: We aim to test whether the machine has a generation rate of $\lambda = 36$, as hypothesized by our colleague. To do this, we will perform a hypothesis test on the parameter mean, comparing the hypothesized rate to the rate of the machine we purchased.

   The mean of the hypothesized machine is $\mu = 36$, because for a Poisson distribution, the mean equals the rate. The observed mean from our data is:

   $$\bar{x} = \frac{3500}{100} = 35.$$

   To conduct the Z-test, we also need the true variance of the Poisson distribution, which is equal to the rate. Therefore, the variance is:

   $$\sigma^2 = 36, \quad \sigma = \sqrt{36} = 6.$$

The standard deviation of the sample is:

$$s = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{100}} = 0.6.$$

We now calculate the Z-test statistic:

$$Z = \frac{x - \mu}{s} = \frac{x - 36}{0.6}.$$

Since we are testing for equality, the test is two-tailed. Thus, we divide the significance level α by 2:

$$\frac{\alpha}{2} = 0.025.$$

From the Z-table, the critical values corresponding to this significance level are $Z = \pm 1.96$. We fail to reject the null hypothesis if:

$$-1.96 < \frac{x - 36}{0.6} < 1.96.$$

Rearranging for x, we obtain the range:

$$34.824 < x < 37.176.$$

The observed mean $\bar{x} = 35$ falls within this range. Therefore, we cannot reject the hypothesis that the machine has a rate of $\lambda = 36$.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361

Figure 1: Two-sided z table.