Quiz 6 - Solution

1. For $0 < x_i < 1$ $(i = 1, 2, \ldots, n)$, the joint p.d.f. $f_n(x|\theta)$ of $X_1, X_2, \ldots, X_n$ is as follows:

$$f_n(x|\theta) = \theta^n \left(\prod_{i=1}^{n} x_i \right)^{\theta-1} .$$

Furthermore, if at least one value of x_i is outside the interval $0 < x_i < 1$, then $f_n(x|\theta) = 0$ for every value of $\theta \in \Theta$. The right side of the above equation depends on x only through the value of the product $\prod_{i=1}^{n} x_i$. Therefore, if we let $h(x) = 1$ and $g(x) = \prod_{i=1}^{n} x_i$, then $f_n(x|\theta)$ can be considered to be factored in the form specified by the factorization theorem. It follows from the factorization theorem that the statistic $T = \prod_{i=1}^{n} X_i$ is a sufficient statistic for θ.