Time: 15 mins
Name: **Std. Number:**

Quiz 1

1. Determine if each of the following functions can be a valid cumulative distribution function (CDF). For each function, provide a brief explanation of why the function does or does not qualify.

   (a) $F(x) = \frac{0.5}{1+e^{-x}}$
   Solution:
   - This function is non-decreasing and satisfies $F(x) = 0$ as $x \to -\infty$, but it fails to reach 1.
   - **Conclusion:** Not a valid CDF.

   (b) $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.5 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$
   Solution:
   - The function is not right-continuous at $x = 1$.
   - **Conclusion:** Not a valid CDF.

   (c) $F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x^2 & \text{if } 0 < x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$
   Solution:
   - The function is non-decreasing and smoothly transitions from 0 to 1.
   - **Conclusion:** Valid CDF.

   (d) $F(x) = 0.9 + 0.1\sin(x)$
   Solution:
   - This function is not non-decreasing.
   - **Conclusion:** Not a valid CDF.

2. Using n samples from the joint distribution of X and Y, provide an estimate for the covariance function $\text{Cov}(X, Y)$. Explain why this estimate is accurate by applying the Law of Large Numbers.

   Solution: To estimate the covariance $\text{Cov}(X, Y)$, use n samples from the joint distribution of X and Y:

$$\widehat{\text{Cov}}(X, Y) = \frac{1}{n} \sum_{i=1}^{n} \left(X_i - \overline{X}\right) \left(Y_i - \overline{Y}\right)$$

where $\overline{X}$ and $\overline{Y}$ are the sample means of X and Y, respectively.

Justification: By the Law of Large Numbers, as $n \to \infty$, the sample means $\overline{X}$ and $\overline{Y}$ converge to the true expected values $\mathbb{E}[X]$ and $\mathbb{E}[Y]$, and the sample covariance $\widehat{\text{Cov}}(X, Y)$ converges to the true covariance $\text{Cov}(X, Y)$. Thus, the estimate becomes more accurate with larger n.