Stochastic Processes September 24, 2024
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Time: 20 mins

Name: Std. Number:

Quiz 0

1. Imagine you're in a stadium where each audience member screams independently of the others. Let X_i represent the volume level of the i-th person's scream, modeled as an independent and identically distributed (IID) random variable with mean μ and variance σ^2. Explain what happens to the average noise level and its variability in the stadium as the number of screaming audience members increases.

   Solution: Each audience member's scream is modeled as an IID random variable X_i with mean μ and variance σ^2. By the **Law of Large Numbers**, as the number of audience members n increases, the average scream volume $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ converges to the population mean μ.

   The variance of the average noise level is $\frac{\sigma^2}{n}$, so as n increases, the variability decreases, meaning the average noise level becomes more predictable and closer to μ.

2. In Switzerland, people have won the highest number of Nobel Prizes per capita, and they also consume the most chocolate per capita in the world. Does this imply that eating more chocolate causes people to win more Nobel Prizes? What is their relation?

   Solution:

   No, higher chocolate consumption does not imply a causal relationship with Nobel Prize wins. This is an example of **correlation**: let X represent chocolate consumption and Y represent Nobel Prize wins. While $\text{corr}(X, Y) \neq 0$, this does not imply **causation** $X \to Y$.

   A third variable, such as economic wealth or education (Z), could influence both X and Y, creating the correlation without direct causality between them. Hence, correlation $\text{corr}(X, Y)$ does not imply causation.

3. If conditioning on event A increases the probability of event B, does conditioning on event B also increase the probability of event A ?

   Solution:

   Yes, If $P(B|A) > P(B)$, then:

   $$\frac{P(B|A)}{P(B)} > 1$$

   By **Bayes' Theorem**:

   $$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

   Since $\frac{P(B|A)}{P(B)} > 1$, this means $P(A)$ is multiplied by a value greater than 1, so $P(A|B) > P(A)$, making A more likely given B.

4. An item has n parts, each with an exponentially distributed lifetime with mean $1/\lambda$. If the failure of one part makes the item fail, what is the average lifetime of the item?

$$\left(X \sim \mathrm{Exp}(\lambda) : F_X(x) = 1 - e^{-\lambda x} \quad f_X(x) = \lambda e^{-\lambda x} U(x) \quad E(X) = \frac{1}{\lambda}\right)$$

Solution: Let X_i be the lifetime of the i th part. The time until the item fails is random variable $X = \min(X_1, X_2, \ldots, X_n)$. Let F be the distribution function of X. We have that

$$\begin{aligned} F(t) &= P(X \le t) = 1 - P(X > t) = 1 - P(X_1 > t, X_2 > t, \ldots, X_n > t) \\ &= 1 - P(X_1 > t) P(X_2 > t) \ldots P(X_n > t) = 1 - e^{-\lambda_1 t} e^{-\lambda_2 t} \ldots e^{-\lambda_n t} \\ &= 1 - e^{-(\lambda_1 + \lambda_2 + \ldots + \lambda_n) t}, \quad t > 0 \end{aligned}$$

Thus X is exponential with parameter $\lambda_1 + \lambda_2 + \ldots + \lambda_n$. But we know that all parameters are the same, thus X is exponential with parameter $n\lambda$ and the average life of the item is $\frac{1}{n\lambda}$.