In the name of GOD.

Sharif University of Technology

Stochastic Process

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Hamid R. Rabiee

Homework 2 Stationary Stochastic Processes, Stochastic Analysis of Systems Deadline : 1403/08/04

1. Let the random process $Y(t)$ be defined as follows, where $X(t)$ is a WSS random process and τ_1 is a fixed delay.

$$Y(t) = X(t + \tau_1) - \alpha X(t)$$

   (a) Find $R_{yy}(t_1, t_2)$ and prove whether $Y(t)$ is a WSS random process.

   (b) Find the cross covariance $C_{xy}(t_1, t_2)$ of $Y(t)$ and $X(t)$. Are theses processes jointly WSS? Prove your answer.

2. Consider the moving average process defined as follows.

$$Y(n) = [X(n) + X(n-1)], X(0) = 0$$

   Where $X(n)$ is a Bernoulli random process.

   (a) Find mean and variance of $Y(n)$ in terms of $P(1) = p$.

   (b) Find the autocorrelation function $R_{yy}(k)$ of $Y(n)$ in terms of p.

3. Consider a WSS random sequence $X[n]$ with mean function μ_X, a constant, and correlation function $R_{XX}[m]$. Form a random process as

$$X(t) = \sum_{n=-\infty}^{\infty} X[n] \frac{sin\pi(t - nT)/T}{\pi(t - nT)/T}$$

   (a) Find $\mu_X(t)$ in terms of μ_X.

   (b) Find $R_{XX}(t_1, t_2)$ in terms of $R_{XX}[m]$. Is $X(t)$ WSS?

4. Consider the random process $X(t)$ where we start from $-\infty$ and continue to ∞, the value of the process flips back and forth between -1 and 1. The switching times are dedicated by a Poisson distribution.

   (a) Find the mean and autocorrelation function of $X(t)$. Is it WSS?

   (b) Find the power spectral density of the random process and average power of it.

5. Given the random process $X(t) = 10cos(100t + \theta)$ where θ is uniformly distributed over $(-\pi, \pi)$. Prove that $X(t)$ is correlation-ergodic, i.e.

$$lim_{T->\infty} \frac{1}{2T} \int_{-T}^{T} X(t)X(t + \tau)dt = R(\tau)$$