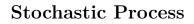
In the name of GOD.



Fall 2024 Hamid R. Rabiee

Homework 2 Stationary Stochastic Processes, Stochastic Analysis of Systems Deadline : 1403/08/04

1. Let the random process Y(t) be defined as follows, where X(t) is a WSS random process and τ_1 is a fixed delay.

$$Y(t) = X(t + \tau_1) - \alpha X(t)$$

- (a) Find $R_{yy}(t_1, t_2)$ and prove whether Y(t) is a WSS random process.
- (b) Find the cross covariance $C_{xy}(t_1, t_2)$ of Y(t) and X(t). Are theses processes jointly WSS? Prove your answer.
- 2. Consider the moving average process defined as follows.

$$Y(n) = [X(n) + X(n-1)], X(0) = 0$$

Where X(n) is a Bernoulli random process.

- (a) Find mean and variance of Y(n) in terms of P(1) = p.
- (b) Find the autocorrelation function $R_{yy}(k)$ of Y(n) in terms of p.
- 3. Consider a WSS random sequence X[n] with mean function μ_X , a constant, and correlation function $R_{XX}[m]$. Form a random process as

$$X(t) = \sum_{n=-\infty}^{\infty} X[n] \frac{\sin\pi(t-nT)/T}{\pi(t-nT)/T}$$

- (a) Find $\mu_X(t)$ in terms of μ_X .
- (b) Find $R_{XX}(t_1, t_2)$ in terms of $R_{XX}[m]$. Is X(t) WSS?
- 4. Consider the random process X(t) where we start from $-\infty$ and continue to ∞ , the value of the process flips back and forth between -1 and 1. The switching times are dedicated by a Poisson distribution.
 - (a) Find the mean and autocorrelation function of X(t). Is it WSS?
 - (b) Find the power spectral density of the random process and average power of it.
- 5. Given the random process $X(t) = 10\cos(100t + \theta)$ where θ is uniformly distributed over $(-\pi, \pi)$. Prove that X(t) is correlation-ergodic, i.e.

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} X(t) X(t+\tau) dt = R(\tau)$$



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