11.4 The data $x[n] = A + w[n]$ for $n = 0, 1, \ldots, N-1$ are observed. The unknown parameter A is assumed to have the prior PDF

$$p(A) = \begin{cases} \lambda \exp(-\lambda A) & A > 0 \\ 0 & A < 0 \end{cases}$$

where $\lambda > 0$, and $w[n]$ is WGN with variance σ^2 and is independent of A. Find the MAP estimator of A.

4)

$$g(A) = p(\underline{x} \mid A)\, p(A)$$

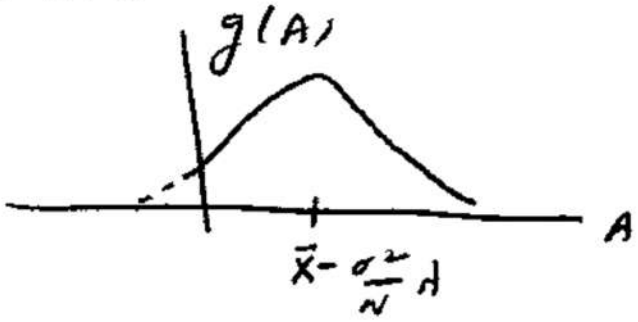
$$= \begin{cases} \dfrac{1}{(2\pi\sigma^2)^{N/2}}\, e^{-\frac{1}{2\sigma^2}\sum_n (x(n)-A)^2}\, \lambda e^{-\lambda A} & A > 0 \\ 0 & A < 0 \end{cases}$$

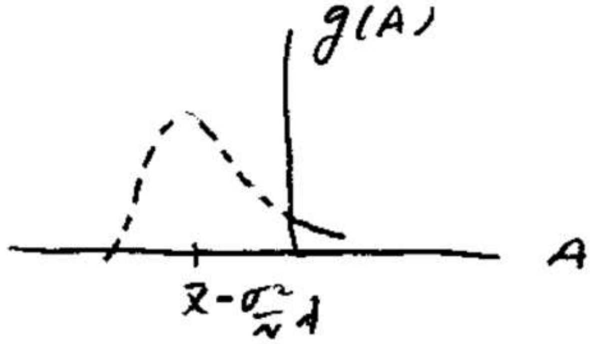
$$\frac{\partial \ln g}{\partial A} = \frac{1}{\sigma^2} \sum_n (x(n) - A) - \lambda = 0$$

$$\Rightarrow \hat{A} = \bar{x} - \frac{\sigma^2}{N}\lambda$$

Note that if $\hat{A}$ is negative we use $\hat{A} = 0$

Since





$$\Rightarrow \quad \hat{A} = \max\left(0, \bar{x} - \frac{\sigma^2}{N}\lambda\right)$$