In the name of GOD.

Stochastic Process

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Practical Homework 1 — Stochastic Processes and Stationary Stochastic Processes — Deadline : 5th Azar

1. (Poisson Process) There are two methods to generate a Poisson process. In this exercise we want to compare these two methods.
   (a) Write function *Poisson1* to simulate the sequence of arrival times of events on $[0, 1]$ for a Poisson process with rate λ, where λ is the input to your functions. The simulation is based on generating i.i.d. inter-arrival times.
   (b) Now write function *Poisson2* to generate the same sequence. in this function you first generate the total number of arrivals and then the conditional distribution of arrival times.
   (c) Set $\lambda = 10$. Run *Poisson1* and *Poisson2* for 10000 rounds, record the total number of arrivals in each round. Plot the empirical distribution of the simulated number of arrivals, and validate your codes by comparing the empirical distribution with the theoretic distribution.
   (d) Compare results from *Poisson1* and *Poisson2*.

2. (Gaussian Process) In this assignment, you will implement a $1D$ Gaussian process that predicts outputs based on noisy training data. You will be given (noisy) $1D$ training data pairs $D_{train} = \{(x_1, y_1), (x_2, y_2)...\}$. Your task is to predict the output for a set of test queries $D = \{\hat{x}_1, \hat{x}_2, ...\}$, conditioned on the training data. The assignment requires you to implement two separate kernel functions, namely the:

   - **Squared Exponential Kernel:**

   $$k(x_i, x_j) = \sigma_f^2 exp\left(-\frac{(x_i - x_j)^2}{2l^2}\right) \tag{1}$$

   where σ_f is a scale factor for the kernel and l is the length scale of the kernel.

   - **Matern Kernel:** This kernel is used commonly in many machine learning applications.

   $$k(x_i, x_j) = \frac{2^{1-\nu}}{\Gamma(\nu)}\left(\frac{\sqrt{2\nu}r}{l}\right)^{\nu} K_\nu\left(\frac{\sqrt{2\nu}r}{l}\right) \tag{2}$$

   where ν and l are (positive) parameters of the kernel and $r = |x_i - x_j|$. K_ν is a modified bessel function and Γ is the gamma function. Good parameters settings for ν are $0.25 - 3$.

   The code for this section is in the file **gp1d.py**.

   (a) Complete the code based on these kernels.
   (b) How the Gaussian process mean and uncertainty changes as more training points are given?
   (c) Play with the kernel parameters to see their effect on the GP.