
Time: 20 mins

Name:

Std. Number:

Quiz 7

Questions

1. Write True if the statement is true; otherwise, write False. [2 Pts.]
 - (a) The UMVUE is an unbiased estimator with minimum MSE among all unbiased estimators.
 - (b) The Fisher information function has to be a decreasing function of the underlying parameter.
2. Let X and Y be independent Poisson-distributed random variables with means $a\theta$ and $b\theta$, where a and b are known and θ is unknown. Find the UMVUE for θ . [8 Pts.]

Solution:

1. (a) True
(b) False
2. The joint density of (X, Y) is given by:

$$\begin{aligned} P(x, y | \theta) &= \frac{(a\theta)^x e^{-(a\theta)}}{x!} \times \frac{(b\theta)^y e^{-(b\theta)}}{y!} \\ &= h(x, y)g(\theta)e^{(x+y)\ln\theta} \end{aligned}$$

and so this is a one-parameter exponential family with complete sufficient statistic $X + Y$. Note that $\{X; Y\}$ is sufficient but not complete. By Theorem 7.3.23, the UMVUE is $\frac{(X+Y)}{(a+b)}$ as it is unbiased and a function of the complete sufficient statistic.