In the name of GOD.

Sharif University of Technology

Stochastic Process

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Quiz 4 - solutions | Power Spectrum + Poisson Process | Release : 31 Oct

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1. A random voltage modeled by a white noise process $X(t)$ with power spectral density $\frac{N_0}{2}$ is input to an RC network, which the frequency response of this system is given by :

$$H(\omega) = \frac{1}{jRC\omega+1} \quad \text{, (R and C are constants)}$$

(a) Find the output PSD $S_Y(\omega)$ (30 points)

(solution) We need the input psd and the system response to find out the output psd :

$$S_Y(\omega) = S_X(\omega)|H(\omega)|^2 = \frac{1}{R^2C^2\omega^2+1}S_X(\omega) = \frac{1}{R^2C^2\omega^2+1}\frac{N_0}{2}$$

(b) Find the output auto correlation $R_Y(\tau)$ (20 points)
Hint :

$$\frac{2\alpha}{\alpha^2+\omega^2} \xrightarrow{\mathcal{F}^{-1}} e^{-\alpha|\tau|}$$

(solution) We need to take the inverse Fourier transform :

$$\frac{1}{R^2C^2\omega^2+1}\frac{N_0}{2} = \frac{\frac{1}{R^2C^2}}{\omega^2+\frac{1}{R^2C^2}}\frac{N_0}{2} = \frac{\frac{2}{RC}}{\omega^2+\frac{1}{R^2C^2}}\frac{N_0}{4RC}$$
$$\xrightarrow{\mathcal{F}^{-1}} \frac{N_0}{4RC}e^{\frac{-|\tau|}{RC}}$$

2. Suppose that incoming calls in a call center arrive according to a Poisson process with intensity of 30 calls per hour.

(a) What is the probability that no call received in a 5-minute period? (20 points)
(solution) Let $N(t)$ denotes the number of incomming calls in t minutes. Then N(t) is a Poisson process with intensity $\lambda = \frac{1}{2}$. Now the probability that no call recived in a 50minute period is equal to $P(N_5 = 0) = e^{\frac{-5}{2}}$

(b) what is the probability that more than 12 calls(> 12) are received in a 30-minute interval? (30 points)

(solution) Let p be the probability that more than 12 calls are received in a 30-minute interval. Then :

$$p = P(N30 \geq 13) = \sum_{k=13}^{\infty} e^{-30*\frac{1}{2}} \frac{(30*\frac{1}{2})^k}{k!}$$
$$= e^{-15} \sum_{k=13}^{\infty} \frac{(15)^k}{k!}$$