Time: 20 mins

Name: Std. Number:

Quiz 2 (Stochastic Processes)

Questions

1. Which of the followings can be autocorrelation function of a WSS process. Prove your answer.
   (a) $R_x(\tau) = \sin(\omega\tau)$
   (b) $R_x(\tau) = \cos(\omega\tau)$
   (c) $R_x(\tau) = e^{-|\tau|}$
2. Can a process be ergodic but not stationary? Provide a theoretical justification or example to support your answer.
3. Investigate two WSS processes, $X(t)$ and $Y(t)$, which are independent. Given that their means are μ_x and μ_y and their respective autocorrelations are $R_x(\tau)$ and $R_y(\tau)$, and a new process $Z(t)$ is defined as $Z(t) = X(t)Y(t)$:
   (a) Determine the mean μ_z and the autocorrelation $R_z(t, \tau)$ of $Z(t)$. Subsequently, ascertain if $Z(t)$ qualifies as a WSS process.
   (b) Evaluate if $Z(t)$ and $X(t)$ are jointly WSS. Provide a justification for your answer.

Quiz 2 Solution (Stochastic Processes)

Solutions

1. (a) $R_x(\tau) = \sin(\omega\tau)$
   - Since the function is not even, it cannot be the autocorrelation function of a WSS process.

   (b) $R_x(\tau) = \cos(\omega\tau)$
   - $R_x(0) = \cos(0) = 1$ which is non-negative.
   - $R_x(\tau) = \cos(\omega\tau)$ and $R_x(-\tau) = \cos(-\omega\tau) = \cos(\omega\tau)$ so it is symmetric.
   - The PSD of $R_x(\tau) = \cos(\omega\tau)$ is a pair of impulses at $\pm\omega$, both of which are non-negative. Therefore, $R_x(\tau) = \cos(\omega\tau)$ is non-negative definite.

   (c) $R_x(\tau) = e^{-|\tau|}$
   - $R_x(0) = e^0 = 1$ which is non-negative.
   - $R_x(\tau) = e^{-|\tau|}$ and $R_x(-\tau) = e^{-|-\tau|} = e^{-|\tau|}$ so it is symmetric.
   - $f(\tau) = e^{-|\tau|}$ is always non-negative for all real values of τ, making it non-negative definite.

2. (a)

$$\mu_z = E[Z(t)] = E[X(t)Y(t)] \tag{1}$$

   Since $X(t)$ and $Y(t)$ are independent:

$$\mu_z = \mu_x \mu_y \tag{2}$$

$$\begin{aligned} R_z(t,\tau) &= E[Z(t+\tau)Z(t)] \\ &= E[X(t+\tau)Y(t+\tau)X(t)Y(t)] \\ &= E[X(t+\tau)X(t)]E[Y(t+\tau)Y(t)] \\ &= R_x(\tau)R_y(\tau) \end{aligned} \tag{3}$$

   $Z(t)$ is WSS since its mean μ_z is constant and its autocorrelation $R_z(t,\tau)$ depends only on τ.

   (b) For $Z(t)$ and $X(t)$ to be jointly WSS, both must have stable averages over time, and their combined behavior should only depend on the time gap, τ. While both $Z(t)$ and $X(t)$ meet these conditions individually, their combined behavior is influenced by the relationship between $X(t)$ and $Y(t+\tau)$. If $X(t)$ and $Y(t+\tau)$ aren't independent for some τ, then $Z(t)$ and $X(t)$ might not be jointly WSS.