

Time: 20 mins

Name:

Std. Number:

Quiz 1

Questions

1. Let X and Y be two independent $\text{Uniform}(0,1)$ random variables. Let also $Z=\max(X,Y)$ and $W=\min(X,Y)$. Find $\text{Cov}(Z,W)$.

مسئله یونین ۱

$$Z = \max[X, Y]$$

$$W = \min[X, Y]$$

$$\mathbb{P} F_Z(z) = P(Z \leq z)$$

$$= P(\max(X, Y) \leq z)$$

$$= P((X \leq z) \text{ and } (Y \leq z))$$

$$= P(X \leq z) P(Y \leq z)$$

$$= F_X(z) F_Y(z) \quad \text{چون } X, Y$$

$$\Rightarrow F_Z(z) = \begin{cases} 0 & z \leq 0 \\ z^2 & 0 < z \leq 1 \\ 1 & z > 1 \end{cases}$$

$$\Rightarrow F_Z(z) = \begin{cases} z^2 & 0 \leq z \leq 1 \\ 0 & \text{other} \end{cases} \quad \Rightarrow E[Z] = \int_{-\infty}^{+\infty} z \times 2z \, dz = 2/3$$

$$E[X+Y] = E[X] + E[Y] = 1/2 + 1/2 = 1$$

$$E[X+Y] = E[Z+W] = 1 = E[Z] + E[W]$$

$$\Rightarrow E[W] = 1/3$$

$$\text{Cov}(Z, W) = E[ZW] - E[Z]E[W]$$

$$\begin{aligned} &= \underbrace{E[XY]}_{\substack{\text{چون} \\ X, Y}} - E[Z]E[W] = 1/2 \times 1/2 - 1/3 \times 2/3 \\ &= 1/36 \end{aligned}$$