In the name of GOD.

Sharif University of Technology

Stochastic Process

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Quiz 10 Sampling

1. Use rejection sampling to find Area Under the Curve for the function

$$f(x) = \sin(x)\ ,\ 0 \leq x \leq \frac{\pi}{2}$$

using the samples below

$$(0.540, 0.761), (0.412, 0.624), (1.556, 0.955), (1.106, 0.157), (0.406, 0.860)$$

given that

$$\sin(0.540) = 0.514\ ,\ \sin(0.412) = 0.4\ ,\ \sin(1.556) = 0.999\ ,\ \sin(1.106) = 0.893\ ,\ \sin(0.406) = 0.394$$

Answer

Estimate the integral as

$$\hat{S} = \frac{1}{N}\sum_{i=1}^{N} \mathbb{I}[f(x_i) \geq y_i] = 0.4$$

2. Use gibbs sampling to find the following probability

$$\mathbb{P}[x_1+, x_2+|x_3-, x_4-]$$

given

$$\mathbb{P}[x_1+] = 0.8$$

$$\mathbb{P}[x_2+] = \begin{cases} 0.6 & x_1+, x_3- \\ 0.4 & x_1-, x_3- \end{cases}$$

Construct $(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, x_4^{(1)}), \ldots, (x_1^{(6)}, x_2^{(6)}, x_3^{(6)}, x_4^{(6)})$ using the following uniform random variables

$$0.21, 0.18, 0.84, 0.27, 0.34, 0.09, 0.87, 0.13, 0.93, 0.15$$

And estimate the probability. At each iteration first update x_1 then x_2. Use birn-in to estimate. Start with x_1+, x_2+, x_3-, x_4-.

Answer

Constructed data is as follows

$$(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, x_4^{(1)}) = +, +, -, -$$

$$(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}, x_4^{(2)}) = +, +, -, -$$

$$(x_1^{(3)}, x_2^{(3)}, x_3^{(3)}, x_4^{(3)}) = -, +, -, -$$

$$(x_1^{(4)}, x_2^{(4)}, x_3^{(4)}, x_4^{(4)}) = +, +, -, -$$

$$(x_1^{(5)}, x_2^{(5)}, x_3^{(5)}, x_4^{(5)}) = -, +, -, -$$

$$(x_1^{(6)}, x_2^{(6)}, x_3^{(6)}, x_4^{(6)}) = -, +, -, -$$

Without burn-in the probability is given as

$$\mathbb{P} = 0.5$$

With burn-in the probability is given as

$$\mathbb{P} = 0.4$$