



1. [15] (One real-world application) The World Wide Web is a large graph with N web pages as vertices (v_1, \dots, v_N) . There is a directed edge from a vertex v_i to another vertex v_j , if v_i 's corresponding page has a hyperlink to that of v_j 's. Google has developed an algorithm called Page Rank, to rank the web pages based on validity/importance. Please study about the Page Rank and explain how is it related to Markov Chains. How is it calculated? Why is the theory behind Page Rank calculation always applicable to it?
2. [15] What is the stationary probability of a random walker on an undirected unweighted connected graph with binary edge matrix of E (assuming it is not bi-partite)? Prove your answer.
3. [15] Prove that the expected number of steps to return from a state to itself in a irreducible positive recurrent Markov Chain is equal to the inverse of the stationary probability of that state.
4. [15] Calculate the expected number of random moves a knight will take to return to the corner of a1 starting from the same place in a chess-board of size 4.
5. [25] A lucky box receives a dollar from the player and returns a prize of 1000\$ with a probability! Everyone thinks the machine tosses a coin to decide whether to give the prize, but it actually has a secret mechanism inside! It has a short memory of 5 digits to keep the last 5 randomly selected digits and if the sequence turns out as 11[*anything_but_1*]11, it passes the prize. Everytime a new digit is randomly selected and appended to the previously generated sequence. Knowing the digits are selected based on a uniform distribution, and considering the initial state of the memory to be 00000, calculate:
 - (a) The probability of at least one win in the first 10, 100, and 10000 moves respectively?
 - (b) The average profit of the owner in a long time run?

Please use a computer for your matrix operations.

6. [15] A medical company performs researches to assess the effectiveness of a newly discovered medication over a special disease. 50 diseased people are randomly selected and divided to two groups of 25. One group, names the case group, receive the medication while the other group, named as control group, receive something irrelevant. After a month, the severity of the case group has been observed to be 4.2 with a standard deviation of 1.5 and the other group 5.8 with a standard deviation of 1.8. Can the company conclude that the medication has been effective with a significance level of 0.01? Assess the results using the p-value approach with a significance level of 0.5.