In the name of GOD.

Sharif University of Technology

Stochastic Process

Fall 2023

Hamid R. Rabiee

Homework 4 | Estimation Theory | Deadline : 14 Azar

1. Let $X_1, \dots, X_n$ be iid with pdf
$$f(x \mid \theta) = \frac{1}{\theta}, \quad 0 \leq x \leq \theta, \quad \theta > 0.$$
Estimate θ using both the method of moments and maximum likelihood. Calculate the means and variances of the two estimators. Which one should be preferred and why?

2. $X_1, X_2, ..., X_n$ are iid samples from $\mathcal{N}(\mu, \sigma^2)$. Consider we know σ^2. Compute UMVUE for μ^3.

3. Let $X_1, \dots, X_n$ be iid with one of two pdfs. If $\theta = 0$, then
$$f(x \mid \theta) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
while if $\theta = 1$, then
$$f(x \mid \theta) = \begin{cases} 1/(2\sqrt{x}) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
Find the MLE of θ.

4. $X_1, X_2, ..., X_n$ are iid samples from a distribution with PDF as follows:
$$f_X(x) = \frac{1}{2\theta} \exp(-\frac{|x|}{\theta}) \text{ where } \theta > 0$$
Find the MSE of the following estimator:
$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} |X_i|$$

5. $X_1, X_2, ..., X_n$ are iid samples from a distribution with PDF as follows:
$$f(x \mid \theta) = \frac{\theta}{(1+x)^{\theta+1}}, \quad 0 < \theta < \infty, 0 < x < \infty$$
Find a sufficient statistics for θ.

6. We have a random variable of X with discrete distribution of:
$$p_X(x|\theta) = \begin{cases} \frac{1}{2\theta+1} & \text{if } x \in \{-\theta, -\theta+1, \cdots, \theta-1, \theta\} \\ 0 & \text{otherwise} \end{cases}$$
By considering $\theta \in \mathbb{N}$, find a sufficient statistic using factorization theorem.