In the name of GOD.

Sharif University of Technology

Stochastic Process

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Hamid R. Rabiee

Homework 2	Stochastic Processes and Stationary Stochastic Processes	Deadline : 9 Aban

1. Answer briefly to each of the following questions:
   - Give an example of a natural phenomenon that can be modeled as a SSS process.
   - Give an example of a natural phenomenon that can be modeled as a WSS process.
   - Give an example of a natural phenomenon that can be modeled as a non-WSS process.
   - Is it possible to say that a process is WSS from a sample path of that process?
   - Is every SSS process an i.i.d stochastic process?
2. Determine if the following functions can serve as valid autocorrelation functions and justify your answer:
   - $\sin(\tau)$
   - $e^{-\tau^2}$
3. If $y(t) = x(t+a) + x(t-a)$, Show that:
   - $R_y(\tau) = 2R_x(\tau) - R_x(\tau + 2a) - R_x(\tau - 2a)$.
   - $S_y(\omega) = 4S_x(\omega)sin^2(a\omega)$.
4. Can a process be ergodic but not stationary? Provide a theoretical justification or example to support your answer.
5. A WSS process $X(t)$ with power spectral density $S_X(\omega) = \frac{1}{\omega^2+4}$ is passed through an LTI system with impulse response $h(t) = e^{-2t}u(t)$. Determine the power spectral density of the output.
6. Let the random variable v have a uniform distribution over the interval [0,1]. Consider the two random processes $X(t) = u(t - v)$ and $Y(t) = \delta(t - v)$.
   (a) Describe each of the two processes above by drawing a sample path.
   (b) Calculate the expected values for each of the processes above.
   (c) Calculate the values for $R_{XX}(t_1, t_2)$, $R_{YY}(t_1, t_2)$, and $R_{XY}(t_1, t_2)$.
7. Consider a process $X(t) = B\cos(wt) + A\sin(wt)$, where A and B are independent and uniformly distributed over $[-1, 1]$. Determine the conditions under which $X(t)$ is ergodic in the mean.
8. Consider a sequence $Z_1, Z_2, Z_3, ...$ which is independent and identically distributed (i.i.d). The probabilities are given by:

$$P(Zi = 1) = p$$
$$P(Zi = -1) = 1 - p = q$$

Define the signal X_n as:

$$X_n = \sum_{i=1}^{n} Zi$$

Where $n = 1, 2, ...$ and $X_0 = 0$.

- Determine the mean and variance of $X[n]$.
- Determine the autocorrelation of $X[n]$.