①

$t-2$	$t-1$	P_L
L	L	0.3
L	R	0.4
R	L	0.6
R	R	0.8

→ landed left side with 10^9 jumps

→ crossed the line 10^9 jumps

بهزاد

$S_0 = LL$
$S_1 = LR$
$S_2 = RL$
$S_3 = RR$

$$P = \begin{bmatrix} 0.3 & 0.7 & 0 & 0 \\ 0 & 0 & 0.4 & 0.6 \\ 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 \end{bmatrix}$$

Stationary:

$\pi_1 = 0.3\pi_1 + 0.6\pi_3 \quad (1)$

$\pi_2 = 0.7\pi_1 + 0.4\pi_3 \quad (2)$

$\pi_3 = 0.4\pi_2 + 0.8\pi_4 \quad (3)$

$\pi_4 = 0.6\pi_2 + 0.2\pi_4 \quad (4)$

$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \quad (5)$

$(1) \rightarrow 0.7\pi_1 = 0.6\pi_3 \quad (6)$

$(4) \rightarrow 0.8\pi_4 = 0.6\pi_2 \quad (7)$

$(6,2) \rightarrow \pi_2 = 0.6\pi_3 + 0.4\pi_3 = \pi_3 \quad (8)$

$5,6,7,8 \rightarrow \frac{6}{7}\pi_3 + \pi_3 + \pi_3 + \frac{6}{8}\pi_3 = 1 \rightarrow \pi_3\left(2 + \frac{6}{7} + \frac{3}{4}\right) = 1$

$$\pi_3 = \frac{28}{56+24+21} = 0.28$$

$\pi_2 = 0.28$

$\pi_1 = 0.24$

$\pi_4 = 20$

a)

$$x_t = \begin{cases} 1 & \text{at on L} \\ 0 & \text{o.w} \end{cases}$$

$$p(x_t) = \begin{cases} p(1) = \pi_1(0.3) + \pi_2(0.4) + \pi_3(0.6) + \pi_4(0.8) = 0.51 \\ p(0) = 1 - p(1) = 0.49 \end{cases}$$

$\rightarrow 10^9(0.51)$

(2)

$$x_t = \begin{cases} 1 & t \text{ crossed line} \\ 0 & \text{o.w.} \end{cases}$$

$$p(1) = \pi_1(0.7) + \pi_2(0.4) + \pi_3(0.4) + \pi_4(0.8) = 0.55$$

$$10^9\,(0.55)$$

$p[x_n = i \text{ for infinitly } n] = 0$

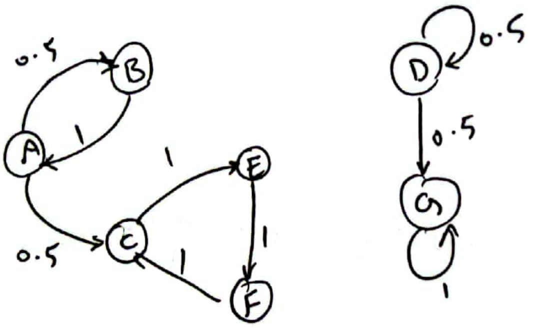
transient نتونیم دوباره برگردیم

recurrent برمی‌گردیم

infinitly often

$p_i[X_n = i \text{ infinitly meinyn}] = 1$

	A	B	C	D	E	F	G
A	-	1					
B	0.5					1	
C	0.5		1	0.5			
D	-						
E		-	1			1	
F				0.5			1
G	-	-	-	-			



1) D , A, B $\rightarrow$ transient

G , C, E, F $\rightarrow$ recurrent

2) period of each state

Period is largest integer d

$p_{ii}^{(n)} = 0$ when $n \,\%\, d \neq 0$

$\forall n \; p_{ii}^{(n)} > 0 \rightarrow d(i) = \infty$

$d_i > 1$ periodic

$d_i = 1$ aperiodic

$d(A) = 2$

$d(B) = 2$

$d(C) = d(E) = d(F) = 3$

$d(D) = d(G) = 1$

③

3) Expected # visits for each transient state Start from A

$$\mathbb{E}[N_A] = 1 + \sum_{n=2}^{\infty} n(0.5)^{n-1}$$

$$\sum_{n=0}^{\infty} p^n = \frac{1}{1-p} \qquad p < 1$$

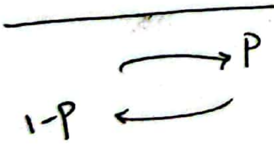
$$\sum_{n=1}^{\infty} n p^{n-1} = \frac{d}{dp}\left[\sum_{n=0}^{\infty} p^n\right] = \frac{d}{dp}\left[\frac{1}{1-p}\right] = \frac{1}{(1-p)^2}$$

$$\Rightarrow \sum_{n=2}^{\infty} n p^{n-1} = \frac{1}{(1-p)^2} - 1$$

$$\Rightarrow \mathbb{E}[N_A] = 1 + \frac{1}{(1-(0.5))^2} - 1 = \boxed{4}$$

$$\mathbb{E}[N_B] = \sum_{n=1}^{\infty} n(0.5)^n = (0.5)\sum_{n=1}^{\infty} n(0.5)^{n-1} = 0.5\,\frac{1}{(1/2)^2} = \boxed{2}$$

$$\mathbb{E}[N_D] = 0$$



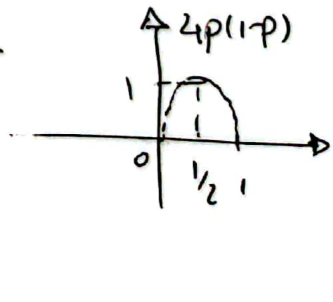
based on p is state recurrent or transient.

$$P_{00}^{2k} = \binom{2k}{k} p^k (1-p)^k = \frac{2k!}{k!\,k!} p^k (1-p)^k$$

Stirling's Approx

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

$$= \frac{\sqrt{2\pi 2k}\left(\frac{2k}{e}\right)^{2k}}{(\sqrt{2\pi k})^2\left(\frac{k}{e}\right)^{2k}} p^k (1-p)^k = \frac{1}{\sqrt{\pi k}}\left(4p(1-p)\right)^k$$



$$\text{if } p = 1/2 \rightarrow \frac{1}{\sqrt{\pi}}\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}} > \frac{1}{\sqrt{\pi}}\sum_{k=1}^{\infty} 1/k = \infty$$

(4)

if $p \neq 1/2 \rightarrow 4p(1-p) = \theta < 1$

$\sqrt{k} \geq 1$

$$\frac{1}{\sqrt{\pi k}}\,\theta^k \rightarrow \sum_{k=1}^{\infty} p_{00}^{(2k)} = \frac{1}{\sqrt{\pi}} \sum_{k=1}^{\infty} \frac{\theta^k}{\sqrt{k}} \leq \frac{1}{\sqrt{\pi}} \sum_{k=1}^{\infty} \theta^k < \infty$$

transient است

شهودی :

$$x_t = \begin{cases} 1 & \text{در گام } t \text{ به سمت جلو برداشته شود} \\ -1 & \text{در گام } t \text{ رو به عقب برداشته شود} \end{cases}$$

$$Y = \sum_{t=1}^{N} x_t = N\,E[x_t] = N(2p-1)$$

$p \neq 1/2 \quad \lim_{N \to \infty} |N(2p-1)| = \infty$ transient

$p = 1/2 \quad \lim_{N \to \infty} |N(0)| = 0$ recurrent

theorem

$$\sum_{n=1}^{\infty} p_{ii}^{(n)} = \infty \rightarrow \text{recurrent}$$

[illegible]

$$\sum_{n=1}^{\infty} p_{ii}^{(n)} < \infty \rightarrow \text{transient}$$