In the name of GOD.
Sharif University of Technology
Stochastic Processes CE 695 Fall 2022 Hamid R. Rabiee

Homework 5 (Estimation Theory)

1. Use method of moments to estimate the parameters μ and σ for the density

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{(x-\mu)^2}{2\sigma^2}\} \tag{1}$$

   based on a random sample $X_1, ..., X_n$

2. Find MLE estimator for the following pdfs. ($X_1, ..., X_n$ is seen)

   (a)

$$f(x|\theta) = \begin{cases} \frac{1}{\theta} & , 0 \leq x \leq \theta \\ 0 & , o.w. \end{cases}$$

   (b)

$$f(x|\theta) = \begin{cases} \frac{1}{\theta} & , 0 < x < \theta \\ 0 & , o.w. \end{cases}$$

   (c)

$$f(x|\theta) = \begin{cases} 1 & , \theta \leq x \leq \theta + 1 \\ 0 & , o.w. \end{cases}$$

   (d)

$$f(x|\theta) = \begin{cases} \frac{1}{\theta_2 - \theta_1} & , \theta_1 \leq x \leq \theta_2 \\ 0 & , o.w. \end{cases}$$

3. Consider n iid samples $x_1, ..., x_n$ drawn from

$$f(x|a) = \frac{1}{a} \quad for x \in [0, a] \tag{2}$$

$$f(x|\eta) = \frac{1}{\eta} exp(-\frac{x}{\eta}) \quad for x > 0 \tag{3}$$

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} exp[-\frac{(x-\mu)^2}{2\sigma^2}] \tag{4}$$

   (a) Derive MLE estimator for each of the pdfs.
   (b) Show that each of $\hat{a}_{ML}, \hat{\eta}_{ML}, \hat{\mu}_{ML}$ is biased or unbiased.
   (c) Show that $\hat{\sigma}^2_{ML}$ is biased and find the unbiased estimator.

(d) Show that MSE of biased estimator for σ^2 is lower than its unbiased one.

4. Let $X_1, ..., X_n$ be iid with pdf $f(x|\theta) = \frac{1}{2\theta}$ with $-\theta < x < \theta$. Find the best unbiased estimator of θ.

5. Let $X_1, ..., X_2$ be iid from below distributions. Is there a function of θ which there exists an unbiased estimator whose variance attains Cramer-Rao lower bound?

   (a)
$$f(x|\theta) = \theta x^{\theta-1} \quad 0 < x < 1, \theta > 0 \tag{5}$$

   (b)
$$f(x|\theta) = \frac{log(\theta)}{\theta - 1} x^{\theta} \quad 0 < x < 1, \theta > 1 \tag{6}$$