In the name of GOD.
Sharif University of Technology
Stochastic Processes CE 695 Fall 2022 H.R. Rabiee

Homework 3 (LTI + Ergodicity)

1. Short answer:

   (a) Is sum of two WSS processes always a WSS process?

   (b) Assume a stochastic process such that $X(t) = At + b$. We know that b is a constant and $A \sim Normal(0, 1)$. Is this process mean ergodic?

   (c) We have a stochastic process that is zero mean and $S_X(\omega) = \frac{5}{9+\omega^2}$. Is this process mean ergodic?

2. Suppose we have an LTI system with the impulse response like below:

$$|H(f)| = \begin{cases} \sqrt{1 + 4\pi^2 f^2} & |f| < 2 \\ 0 & \text{otherwise} \end{cases}$$

   Assume X(t) is a zero mean WSS process such that:

$$R_X(\tau) = e^{-|\tau|}$$

   If X(t) is the input and Y(t) is the output, calculate below parameters:

$$\mu_Y(t)$$

$$R_Y(\tau)$$

$$\mathbb{E}[Y^2(t)]$$

3. Suppose X(t) is a WSS stochastic process and we know that:

$$\mathbb{E}[X(t)] = \mu_X$$

$$R_X(\tau) = e^{-|\tau|}$$

   Assume that A is an independent Gaussian random variable such that:

$$A \sim Normal(\mu_A, \sigma_A^2)$$

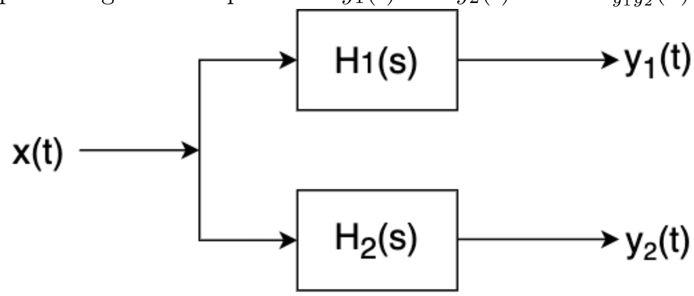
   If we define Y(t) as below:

$$Y(t) = X(t) + A$$

   Prove that :

$$\sigma_A^2 = \mu_X^2 \Leftrightarrow Y(t) \text{ is mean ergodic}$$

4. Let $x(t)$ be a real valued, continuous time, zero mean WSS random process with correlation function $R_{xx}(\tau)$ and power spectrum $S_{xx}(\omega)$. Suppose $x(t)$ is the input to two real valued LTI systems as depicted below, producing two new processes $y_1(t)$ and $y_2(t)$. Find $C_{y_1 y_2}(\tau)$ and $S_{y_1 y_2}(\omega)$



5. Consider an LTI system with system function:

$$H(s) = \frac{1}{s^2 + 4s + 13}$$

The input to this system is a WSS process $X(t)$ with $\mathbb{E}\left[X^2(t)\right] = 10$. Find $S_X(\omega)$ such that the average power of output is maximum.

6. Consider a WSS process $y(t)$ satisfying the equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

where $x(t)$ is a zero-mean WSS process with covariance function:

$$C_{XX}(\tau) = \delta(\tau) + 4e^{-|\tau|}$$

Also assume that transformation from $x(t)$ to $y(t)$ is LTI. Determine $R_{XY}(\tau), S_{XY}(\omega)$.

7. The process $\mathbf{x}(t)$ is WSS with $\mathbb{E}[\mathbf{x}(t)] = 5$ and $R_{xx}(\tau) = 25 + 4e^{-2|\tau|}$. If $\mathbf{y}(t) = 2\mathbf{x}(t) + 3\mathbf{x}'(t)$, find $S_y(\omega)$

8. We have an LTI system. The output of system is defined as:

$$y[n] = x[n] + x[n-2] - \frac{1}{2}x[n-3] + \frac{1}{2}y[n-1]$$

   (a) Find the impulse response of the system in the time and frequency domains.

   (b) Plot the impulse response in time domain from n=0 to n=5.

   (c) If input is a zero-mean WSS process with autocorrelation defined as:

$$R_{XX}[m,n] = R_{XX}[l] = \delta[l] + \delta[|l-1|]$$

   Find the R_{XY}.