In the name of GOD.
Sharif University of Technology
Stochastic Processes CE 695 Fall 2022 H.R. Rabiee

Homework 2 (Stationary Stochastic Processes)

1. We define $X_0 = Y_0$ and $X_n = \lambda X_{n-1} + Y_n$ for $n \geq 1$. Suppose $Y_0, Y_1, ...$ are uncorrelated random variables with $E[Y_n] = 0$ and
$var(Y_n) = \begin{cases} \sigma^2/(1-\lambda^2), & n = 0 \\ \sigma^2, & n \geq 1 \end{cases}$ $where\ \lambda^2 < 1.$
Find $Cov(X_n, X_{n+m})$. Is this process WSS? why?

2. Prove that $Var(X(t+s) - X(t)) = 2R_X(0) - 2R_X(s)$. let X(t) be a WSS.

3. let X_i $(i \in Z)$ be a process in which the X_i's are i.i.d. supposing CDF $F_{X_i}(x) = F(x)$, prove that this process is SSS.

4. Consider the process $X(t) = Y(t + T)$ in which $T \sim U(0, T_0)$ and Y(t) is a periodic function with period T_0. Is this process SSS? Give reasons for your answer.

5. Consider the process $X(t) = Y + Zt$ in wich Y and Z are normal N(1,1) and independent random variables. For this process find the correlation function and covariance function.

6. Define X(t) = A cos(wt) + B sin(wt), where A and B are independent unit normal random variables and w is constant . Show that X(t) is a WSS.

7. Suppose $X_1, X_2, ..$are i.i.d with $E[X_i] = 0$ and $var(X_i) = 4$. For the process $y(n) = X_1 + X_2 + \ldots + X_n (n \in N)$ find the mean and covariance.