In the name of GOD.
Sharif University of Technology
Stochastic Processes CE 695 Fall 2022 Hamid .R. Rabiee

Homework 6 (Markov Chains)

1. A little girl named Sarah loves making origami flowers. It's one of the three indoor activities that she loves to do almost equally. But when the weather is good, she prefers to go outside triple times as staying indoors. Each day Sarah makes origami flowers, she gives one flower to her mom. Assuming the weather condition stays the same as yesterday with a probability of 0.7:

   (a) Build a Markov chain representing the problem and calculate its required probabilities.

   (b) Calculate the expected number of flowers Sarah's mom has gathered in 10 years (yep she had continued her routine in 10 years!)

   (c) This Saturday the weather was good. With what probability Sarah's mom will receive two flowers by Monday night (in 3 days)?

   (d) If Sarah's mom received only one flower on Sunday (and none in Saturday and Monday), what had been the most probable weather conditions for Sunday and Monday?

2. A traveler travels from town to town and never stops! The probability of traveling into each town based on the previous location is presented in the following table. Assuming the traveler is now in Bangs, what is the probability of visiting Fries twice in four next travels without visiting Bluff and Cool? (Highly recommended! Do the computations with a computer!)

States	Bangs	Speed	Fries	Bluff	Cool
Bangs	0	0.1	0.3	0.4	0.2
Speed	0.2	0	0.2	0.1	0.5
Fries	0.2	0.2	0	0.2	0.2
Bluff	0.2	0.4	0.2	0	0.2
Cool	0.3	0.5	0.1	0.1	0

3. Suppose we have a random DNA-like sequence generator! DNA is only composed of A, C, G, and T letters. Our generator simply randomizes the next letter based on the current one. The generation probabilities are presented in the following table.Calculate the probability of observing the following patterns in two states: Starting from letter T and in a random location in long time generation:

   - TTAC

States	A	C	G	T
A	$\frac{1}{2}$	0	$\frac{1}{2}$	0
C	$\frac{3}{4}$	0	$\frac{1}{4}$	0
G	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{8}$	0
T	$\frac{1}{4}$	0	0	$\frac{3}{4}$

- GGCGG

4. Prove that a random walk on an undirected, connected graph is aperiodic if and only if the graph is not bipartite.

5. Show that every Markov chain with $M < \infty$ states contains at least one recurrent set of states.

6.
   - Show that an ergodic Markov chain with M states must contain a cycle with $\tau <$ M states.
   - Let X be a fixed state on this cycle of length τ. Let T(m) be the set of states accessible from X in m steps. Show that:

$$\forall m \geq 1; T(m) \subseteq T(m + \tau) \tag{1}$$